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SPEED/ACCURACY TRADEOFF WHEN USING A LOCK–IN AMPLIFIER TO MEASURE SIGNAL IN THE PRESENCE OF RANDOM NOISE

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DL INSTRUMENTS

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The lock–in amplifier (LIA) operates as a narrowbanding ac voltmeter. It achieves the bandwidth restriction by means of a Time Constant control (T) which sets the cutoff frequency of its lowpass output filter. For most applications, better results are obtained by employing two RC sections connected in cascade (i.e., 12 dB/octave) rather than just one (i.e., 6 dB/octave). This yields faster settling and sharper attenuation of discrete frequency interferences, as explained in IAN 23, *"The Heterodyning Lock–In Amplifier"* and IAN 35, *"The Evolution of the Modern Lock–In Amplifier"*. For the 12 dB/octave mode the equivalent noise bandwidth (B) is related to the Time Constant as follows:

(i) $$B = \frac{1}{8T}, \text{ Hz, equiv. noise bw}$$

The signal indication seen at the output of the LIA (e_s, volts rms) will fluctuate due to random noise at the input which gets through this passband. The input noise is most usefully expressed as an rms voltage normalized to a one Hz bandwidth. It thus represents the *input* spectral noise density (e_n, volts rms/$\sqrt{\text{Hz}}$). This leads to a normalized input signal to noise density (SNDR) definition:

(ii) $$\text{SNDR} = \frac{e_s}{e_n}, \sqrt{\text{Hz}}$$

Knowing SNDR and B, one can readily calculate the uncertainty in the signal measurement due to noise. Expressed as a fractional reproducibility (σ_x), it equals the inverse of the *output* signal to noise ratio (SNR):

(iii) $$\sigma_x = \frac{1}{\text{SNR}} = \frac{\sqrt{B}}{\text{SNDR}} = \frac{e_n}{e_s\sqrt{8T}}, \text{ rms}$$

The value of σ_x expresses the statistical standard deviation of the fluctuations one would observe in the LIA output reading due to source noise. That is, 68% of the time the instantaneous output would lie within $\pm\sigma_x$ of the true value. On a peak–to–peak basis one would see about six times this variation ($6\,\sigma_x$).

The response time to a change in signal level will be directly proportional to the LIA Time Constant. This represents the time penalty you inescapably must pay when extracting a measurement from a noisy system. Any narrowbanding approach demands increasing measurement time as the *square* of the desired accuracy improvement. You have no choice but to increase signal or decrease noise to improve the situation.

The 12 dB/octave LIA output filter will settle to within 1% in 6.6 time constants. For good measure, lets say it takes a measurement time (t_m) of 8T to obtain a reading. We can then rewrite the previous equations as:

(iv) $$t_m = 8T = \left[\frac{e_n}{e_s\,\sigma_x}\right]^2 \text{ seconds}$$

Another way to look at this is to observe that we cannot meaningfully sample the output of the LIA any faster than the Nyquist rate. To do so would lead to auto–correlated readings. This establishes a sampling rate (r_s), beyond which a shorter data acquisition interval yields no further information, equal to twice the noise bandwidth B:

(v) $$r_s = 2B = \frac{1}{4T} = 2\left[\frac{e_s\,\sigma_x}{e_n}\right]^2, \text{ Hz}$$

As an example, let us suppose that the rms input broadband signal to rms noise ratio is –40 dB (SNR = 0.01, or the noise is 100 times larger than the signal) and that the source bandwidth is from 0 to 10 kHz. (Since the peak–to–peak noise is about six times the rms value, this corresponds to a –55 dB SNR on an rms signal to peak–to–peak noise basis.) Assuming the noise spectral density is flat, we have:

e_n = (source broadband noise)/$\sqrt{\text{source bandwidth}}$

from (ii):

(vi) $$\text{SNDR} = \frac{e_s \sqrt{\text{source bandwidth}}}{\text{(source broadband noise)}}$$

For this example:

$$\text{SNDR} = 0.01 \sqrt{10\,\text{kHz}} = 1 \sqrt{\text{Hz}}$$

If we chose the longest available time constant on our lock–in (e.g., T = 125 seconds, B = 0.001 Hz), then from (iii) and (iv):

$$\sigma_x = \sqrt{.001}\,/1 = 0.0316$$

$$t_m = 1000 \text{ seconds} = 17 \text{ minutes}$$

The $\pm 3\sigma$ reproducibility will be about ±10% with a fluctuation "period" on the order of ten minutes.

This example illustrates that for random interference, the availability of extremely high dynamic reserve is superfluous. There is no way you can make use of a 60 dB rms–rms (75 dB pp–rms) dynamic reserve unless you are willing to average measurements over a period of an hour or so to get reasonable accuracy.

NOTE #1 SIGNAL AVERAGING FOR HIGH ACCURACY

Using a faster time constant, then averaging signal samples taken at speeds above the Nyquist limit will result in faster measurement than relying on the LIA time constant alone (e.g., 60% as long to achieve 1% standard deviation.)

NOTE #2 EQUATIONS FOR 6 dB/OCTAVE LIA TIME CONSTANT ROLLOFF

(vii) $$B = \frac{1}{4T}$$

(viii) $$\sigma_x = \frac{1}{\text{SNDR}\sqrt{4T}}$$

The output settles to 1% in 3.9T. Let's assume 5T for good measure. Then:

(ix) $$t_m = 5T = \frac{1}{\text{SNDR}^2 \sigma_x^{\,2}}$$

(x) $$r_s = 2B = \frac{1}{2T} = 2.5\,(\text{SNDR}\,\sigma_x)^2$$